

Huffman Encoding

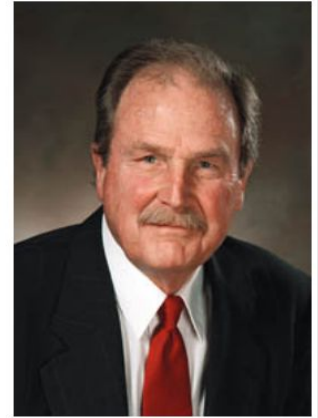
Chad Germany

History

- Huffman was an electrical engineering student of Fano
- In 1951 as a student. Huffman found the most efficient binary code.
- As a result he did not have to take his final.



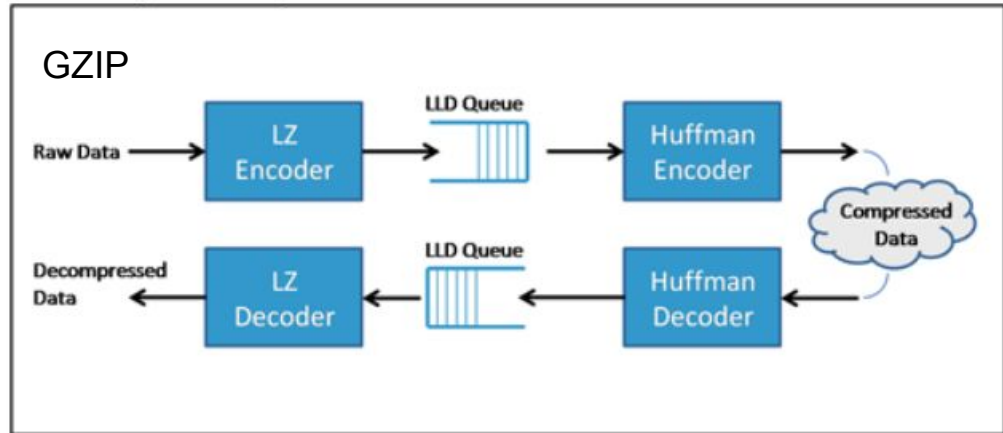
Robert Fano



David A. Huffman

Why is it important ?

- Huffman coding is used in conventional compression formats like GZIP, BZIP2, PKZIP, etc.
 - gzip is based on the DEFLATE algorithm, which is a combination of LZ77 and Huffman coding



<https://en.wikipedia.org/wiki/Gzip>

<https://www.chipestimate.com/Unzipping-the-GZIP-compression-protocol/Altior/Technical-Article/2010/03/23>

What is Huffman encoding

- Huffman Encoding is a technique of compressing data to reduce its size without losing any of the details.
- Huffman Coding is generally useful to compress the data in which there are frequently occurring characters.
- The most frequent character gets the smallest code and the least frequent character gets the largest code
- The variable-length codes assigned to input characters are Prefix Codes
 - [5,9,59] is not a prefix code

Algorithm for Huffman Encoding

1. Create dictionary of characters with frequencies
2. create a priority queue Q consisting of each unique character.
3. sort them in ascending order of their frequencies.
4. for all the unique characters:
 - a. create a newNode
 - b. extract minimum value from Q and assign it to leftChild of newNode
 - c. extract minimum value from Q and assign it to rightChild of newNode
 - d. calculate the sum of these two minimum values and assign it to the value of newNode
 - e. insert this newNode into the tree
5. return rootNode

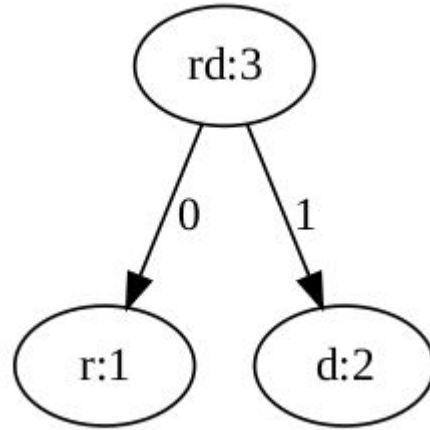
Building a Huffman Tree

Want to encode: **“feed me more food”**

Step 1: Calculate frequency of every character in the text, and order by increasing frequency. Store in a queue which is a minimum heap.

r : 1 | d : 2 | f : 2 | m : 2 | o : 3 | 'SPACE' : 3 | e : 4

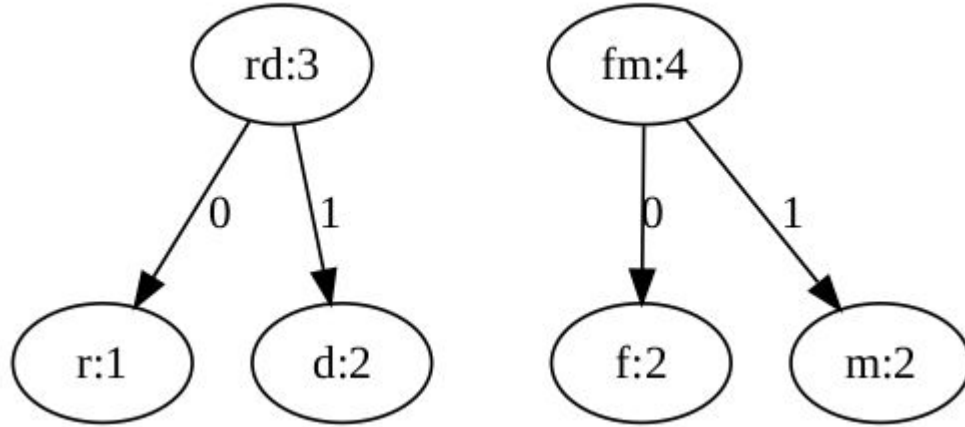
Step 2: Build the tree from the bottom up. Start by taking the two least frequent characters and merging them (create a parent node for them). Store the merged characters in a new queue:



SINGLE: f : 2 | m : 2 | o : 3 | 'SPACE' : 3 | e : 4

MERGED: rd : 3

Step 3: Repeat Step 2 this time also considering the elements in the new queue. 'f' and 'm' this time are the two elements with the least frequency, so we merge them

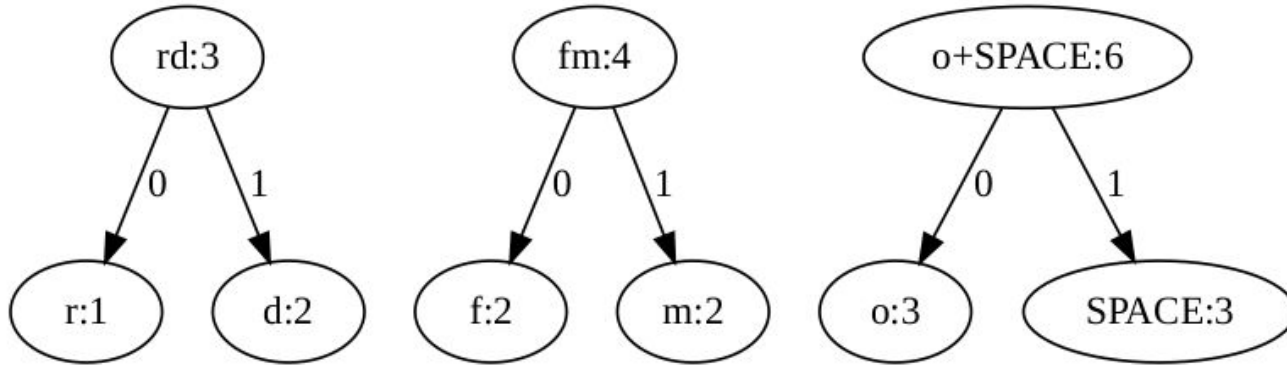


SINGLE: o : 3 | 'SPACE' : 3 | e : 4

MERGED: rd : 3 | fm : 4

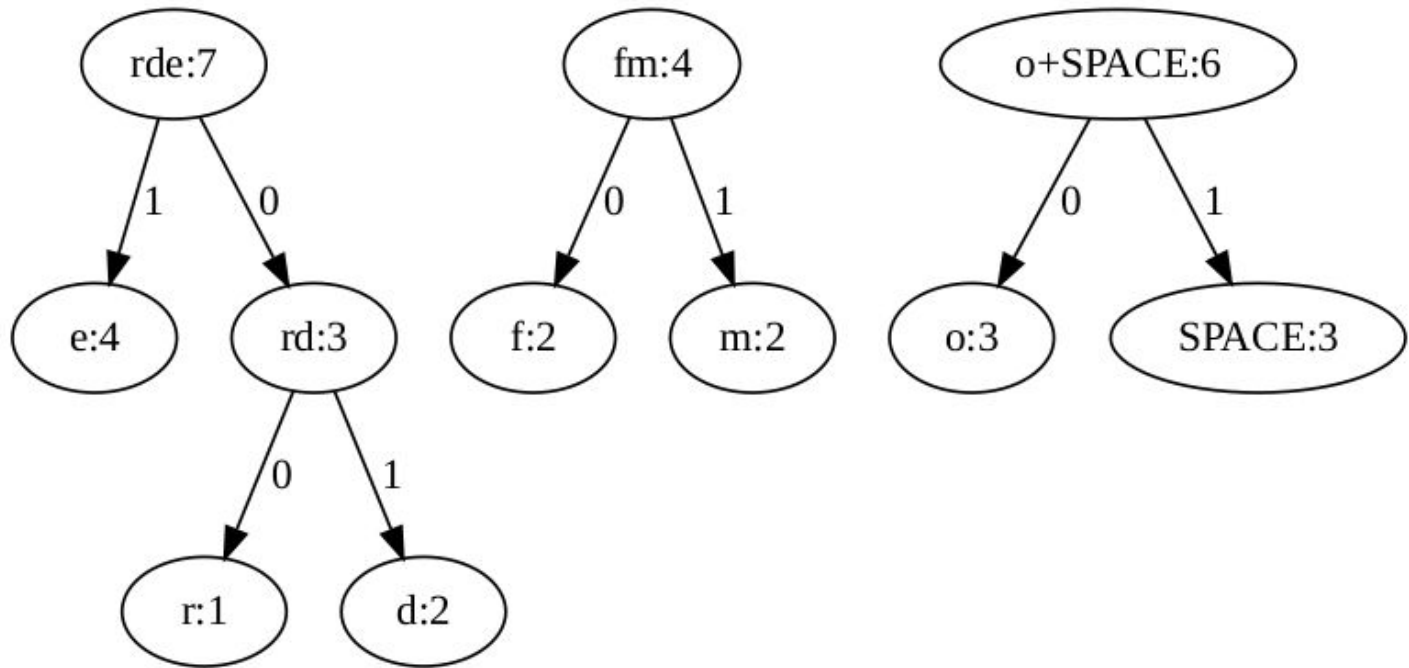
Step 4: Repeat Step 3 until there are no more elements in the SINGLE queue, and only one element in

the MERGED queue:



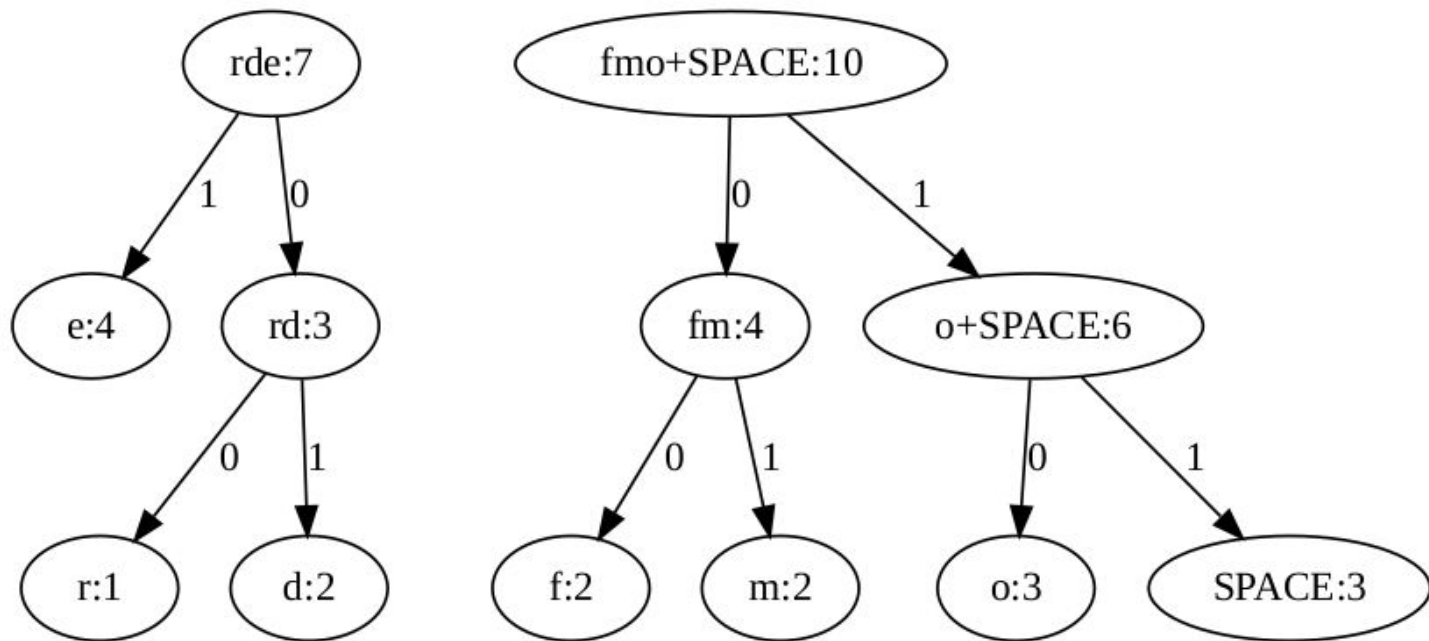
SINGLE: e : 4

MERGED: rd : 3 | fm : 4 | o+SPACE : 6



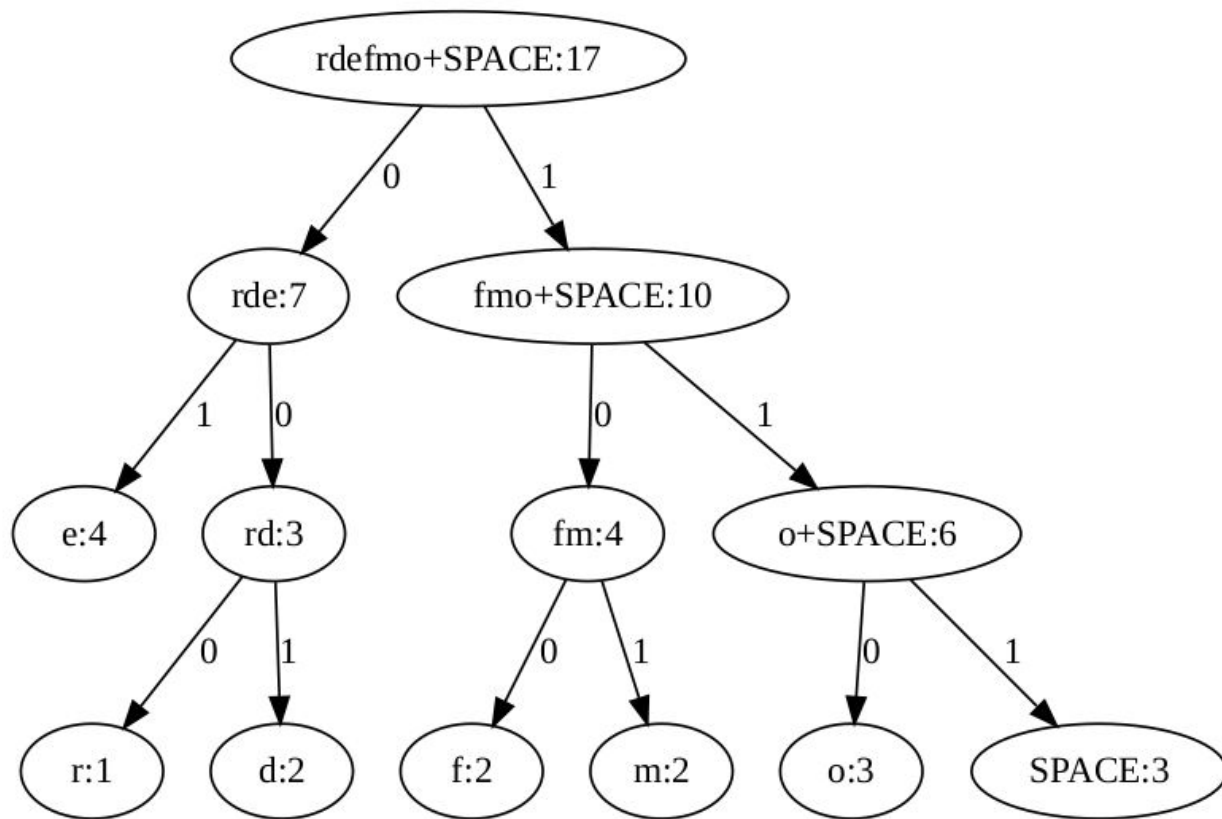
SINGLE:

MERGED: fm : 4 | o+SPACE : 6 | rde: 7



SINGLE:

MERGED: rde: 7 | fmo+SPACE: 10



SINGLE:

MERGED: rdefmo+SPACE: 17

In addition to saving the compress message we need to also save the tree itself.

Algorithm:

- 1) Start at the root
- 2) If the current node is a leaf:
 - a) Write a “1” to the output file
 - b) Write the character that the leaf node represents to the output file
- 3) Else (the current node is an internal node):
 - a) Write a “0” to the output file
 - b) Recurse on the left subtree, then the right subtree

For the tree in the example the code is:

```
001 1e001d1o001r101f1m
```

Huffman Table:

| Character | Frequency | Code | Size |
|-----------|-----------|------|---------|
| e | 4 | 01 | $4*2=8$ |
| | 3 | 00 | $3*2=6$ |
| o | 3 | 101 | $3*3=9$ |
| d | 2 | 100 | $2*3=6$ |
| r | 1 | 1100 | $1*4=4$ |
| \n | 1 | 1101 | $1*4=4$ |
| f | 2 | 1110 | $2*4=8$ |
| m | 2 | 1111 | $2*4=8$ |
| $8*8=64$ | | | 55 |

The total after compression is 119 bits

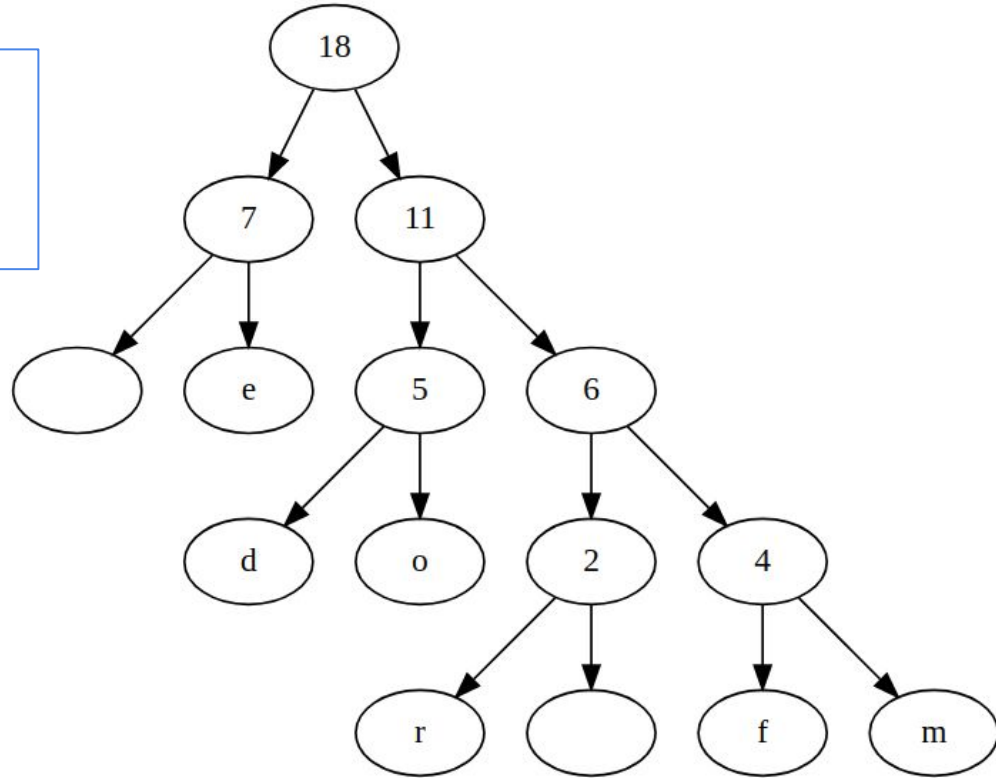
Before compression
 $18*8 = 144$ bits

So we saved 25 bits

Huffman Tree

For the tree in the example the code is:

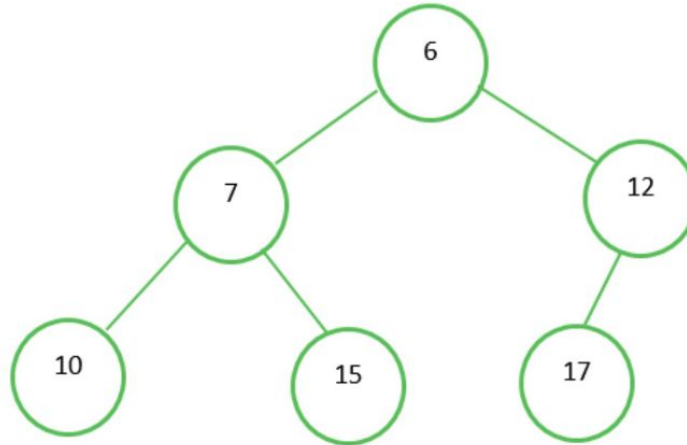
001 1e001d1o001r101f1m



Some words on Minimum Heap

- In a Min-Heap the key present at the root node must be less than or equal to among the keys present at all of its children.
- In a Min-Heap the minimum key element present at the root.
- A Min-Heap uses the ascending priority.
- In the construction of a Min-Heap, the smallest element has priority.
- In a Min-Heap, the smallest element is the first to be popped from the heap.
- For restructuring after popping $O(\log n)$ time complexity

Min-Heap



<https://www.geeksforgeeks.org/difference-between-min-heap-and-max-heap/>

Time Complexity

- To encode message length n , with c possible characters
 - Count frequencies: $O(n)$
 - Build tree: $O(c \log c)$ (with priority queue)
 - Encode: $O(n)$

Resources to read

- <https://www.cs.utoronto.ca/~brudno/csc373w09/huffman.pdf>
- <https://www.chipestimate.com/Unzipping-the-GZIP-compression-protocol/Altior/Technical-Article/2010/03/23>
- <https://en.wikipedia.org/wiki/Lempel%E2%80%93Ziv%E2%80%93Storer%E2%80%93Szymanski>
- <https://www.huffmancoding.com/my-uncle/david-bio> (Ken Huffman's website about his uncle)
- <https://www.maa.org/press/periodicals/convergence/discovery-of-huffman-codes>
- <https://www.geeksforgeeks.org/huffman-coding-greedy-algo-3/>
- <https://courses.cs.washington.edu/courses/cse326/10wi/lectures/lec24/lec24-10wi-Huffman.pdf>
- <https://www.programiz.com/dsa/huffman-coding>
- <https://www.youtube.com/watch?v=fWk6Y8Rd6bs> (youtube video)

Lz77 dictionary

Encoding of the string:
abracadabrad

output tuple: (offset, length, symbol)

| | | | | | | | | | | output | | | |
|-------|---|---|---|---|---|---|---|---|---|--------|---|--------|---------|
| 7 | 6 | 5 | 4 | 3 | 2 | 1 | | | | | | | |
| | | | | | | | a | b | r | a | c | ada... | (0,0,a) |
| | | | | | | a | b | r | a | c | a | dab... | (0,0,b) |
| | | | | | a | b | r | a | c | a | d | abr... | (0,0,r) |
| | | | | a | b | r | a | c | a | d | a | bra... | (3,1,c) |
| | | a | b | r | a | c | a | d | a | b | r | ad | (2,1,d) |
| a | b | r | a | c | a | d | a | b | r | a | d | | (7,4,d) |
| ...ac | a | d | a | b | r | a | d | | | | | | |

| | | |
|---------------|-------------------|---|
| Search buffer | Look-ahead buffer | 12 characters compressed into 6 tuples |
| | | Compression rate: $(12 \cdot 8) / (6 \cdot (5 + 2 + 3)) = 96 / 60 = 1,6 = 60\%$. |